

A NOTE ON THE UNIRATIONALITY OF A MODULI SPACE OF DOUBLE COVERS

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ABSTRACT. In this note we look at the moduli space $\mathcal{R}_{3,2}$ of double covers of genus three curves, branched along 4 distinct points. This space was studied by Bardelli, Ciliberto and Verra in [BCV]. It admits a dominating morphism $\mathcal{R}_{3,2} \rightarrow \mathcal{A}_4$ to Siegel space. We show that there is a birational model of $\mathcal{R}_{3,2}$ as a group quotient of a product of two Grassmannian varieties. This gives a proof of the unirationality of $\mathcal{R}_{3,2}$ and hence a new proof for the unirationality of \mathcal{A}_4 .

1. INTRODUCTION

In this short note we study the moduli space $\mathcal{R}_{3,2}$ of Bardelli-Ciliberto-Verra [BCV]. $\mathcal{R}_{3,2}$ parametrises the following data: triples (C, B, L) , where C is a smooth connected projective curve of genus 3, L is a line bundle of degree 2 on C and B is a divisor in the linear system $|L^2|$, consisting of distinct points. Since a double cover of a genus 3 curve ramified in 4 points has genus 7, this moduli problem may be viewed as parametrising certain generalized Prym varieties of dimension 4 which are quotients of a genus 7 Jacobian [BCV]. The importance of $\mathcal{R}_{3,2}$ comes from the fact that this Prym construction provides a dominant, generically finite morphism $\mathcal{R}_{3,2} \rightarrow \mathcal{A}_4$ to Siegel space, which implies in particular that the generic principally polarised abelian fourfold contains a finite number of curves of the minimal genus 7 [BCV].

In this note we first describe $\mathcal{R}_{3,2}$ as follows:

Theorem 1.1. *The moduli space $\mathcal{R}_{3,2}$ is birational to the group quotient*

$$\mathcal{R}_{3,2} \sim (G(3, U^+) \times G(4, U^-))/H$$

of a product of Grassmannians $G(3, U^+) \times G(4, U^-)$ by a certain subgroup $H \subset SO(10)$ which is contained in the centraliser of the action of an involution i on $SO(10)$. Moreover, there is an irreducible 16-dimensional projective representation U of $SO(10)$ and $U = U^+ \oplus U^-$ is a splitting as \pm -eigenspaces for the involution i acting on U .

This description of $\mathcal{R}_{3,2}$ is similar to the descriptions obtained for the various moduli spaces \mathcal{M}_g , for small $g \leq 9$, by Mukai and others (for example, see [Mk], [Mk2]).

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In another direction, it is of wide interest to know when such moduli spaces are rational or unirational varieties. It is known from the results of Severi, Sernesi, Katsylo, Mukai, Dolgachev, Chang-Ran, Verra that the moduli spaces \mathcal{M}_g , for small $g \leq 14$ are unirational [Se], [Ka], [Do], [Ve], [Ch-Rn]. Some moduli spaces of bielliptic covers have also been shown to be rational by Bardelli-Del Centina [B-dC]. Unirationality of the moduli space of étale double covers of genus 5 curves, \mathcal{R}_5 , is known, due to Izadi-Lo Giudice-Sankaran [Iz-L-S] and Verra [Ve2]. See also a very recent preprint [Fa-Ve].

The above description of $\mathcal{R}_{3,2}$ in Theorem 1.1 says

Theorem 1.2. *The moduli space $\mathcal{R}_{3,2}$ (and hence \mathcal{A}_4) is a unirational variety.*

Although the implication about \mathcal{A}_4 was known before (due to Clemens), our method gives some interesting insight and may have further applications. It would be interesting to study the subgroup H in the theorem and its representations U^+ and U^- in more detail. We pose some open questions in this direction, see 2.9.

The proof of the main theorem proceeds by analysing Mukai's description of the moduli space \mathcal{M}_7 and restricting the attention to the sublocus $\mathcal{R}_{3,2} \subset \mathcal{M}_7$. This sublocus is contained in the singular locus of \mathcal{M}_7 and parametrises curves with an involution. The involution plays a crucial role in determining the Grassmannian varieties, in the statement of Theorem 1.1.

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2. A BIRATIONAL MODEL OF THE MODULI SPACE $\mathcal{R}_{3,2}$ OF BARDELLI-CILIBERTO-VERRA

In this section, we will look at the moduli space $\mathcal{R}_{3,2}$ studied by Bardelli-Ciliberto-Verra [BCV]. More precisely, let $\mathcal{R}_{3,2}$ be the moduli space of all isomorphism classes of double coverings $f : C' \rightarrow C$ with C a smooth curve of genus 3, C' irreducible and f branched at 4 distinct points of C . Alternatively, $\mathcal{R}_{3,2}$ is the moduli space of isomorphism classes of triples (C, B, L) , where C is a smooth curve of genus 3, B is an effective divisor on C formed by 4 distinct points and L is a line bundle on C such that $L^{\otimes 2} \simeq \mathcal{O}(B)$.

Note that the genus of the curve C' is $g' = 7$ and $\mathcal{R}_{3,2} \subset \mathcal{M}_7$ (see [Co, p.138]). Then we have

$$\dim \mathcal{R}_{3,2} = 10.$$

Our main theorem in this section is the following:

Theorem 2.1. *The moduli space $\mathcal{R}_{3,2}$ is birational to the group quotient of a product of Grassmannians $G(3, U^+) \times G(4, U^-)$, by an algebraic subgroup $H \subset SO(10)$. Here H is*

contained in the centraliser of the action of an involution i on $SO(10)$. Moreover, there is an irreducible 16-dimensional projective representation U of $SO(10)$ and $U = U^+ \oplus U^-$ is a splitting as \pm -eigenspaces for the involution i acting on U .

Our proof follows by analysing Mukai's classification [Mk], [Mk2] of the generic genus 7 canonical curve, taking into account the action of the involution. Whenever a genus 7 smooth curve is not tetragonal, then it is a linear section of an orthogonal Grassmannian $X_{10} \subset \mathbb{P}^{15}$, given by the spinor embedding (see [Mk, p.1632]). Here $\mathbb{P}^{15} = \mathbb{P}(U_{16})$ where U_{16} is the irreducible spinor representation of the spin group $Spin(10)$. Hence the space U_{16} is a projective representation of the special orthogonal group $SO(10)$. Projectively, this can be translated to say that the group $SO(10)$ acts on \mathbb{P}^{15} and leaves the orthogonal Grassmannian X_{10} invariant. In particular $SO(10)$ also acts on the linear subspaces of \mathbb{P}^{15} and we will require its action on the Grassmannian $G(7, U_{16})$. This is because a general linear subspace $\mathbb{P}^6 \subset \mathbb{P}^{15}$ restricted to X_{10} gives a canonical curve C of genus 7. In other words, \mathbb{P}^6 is the complete linear system given by the canonical bundle on $C = \mathbb{P}^6 \cap X_{10}$.

Furthermore, we have the following result on the embedding into the homogeneous space.

Theorem 2.2. *Assume that two linear spaces P_1, P_2 cut out smooth curves C_1, C_2 from the symmetric space $X_{10} \subset \mathbb{P}^{15}$ respectively. Then any isomorphism from C_1 onto C_2 extends to an automorphism ϕ of $X_{10} \subset \mathbb{P}^{15}$ with $\phi(P_1) = P_2$.*

Proof. See [Mk2, Theorem 3]. □

This theorem characterises the non-tetragonal curves of genus 7. Explicitly, the moduli space has the following birational model [Mk, §5, p.1639]:

$$\mathcal{M}_7 \sim G(7, U_{16})/SO(10).$$

To obtain a birational model of $\mathcal{R}_{3,2}$, we will utilise the above birational model of \mathcal{M}_7 and analyse the birational equivalence restricted to the sublocus $\mathcal{R}_{3,2}$.

We will need the following lemma in our proof of Theorem 2.1. We say that a curve C' is tetragonal if and only if there is a line bundle $\mathcal{L} \in g_4^1(C')$.

Note that the data $(C, B, L) \in \mathcal{R}_{3,2}$ corresponds to the data (C', i) , where C' is a genus 7 curve with an involution i . Denote the quotient map $f : C' \rightarrow C = C'/\langle i \rangle$.

Lemma 2.3. *With notations as above, consider a double cover $f : C' \rightarrow C$, defined by a line bundle L branched along the set B of 4 distinct points, and such that $L^2 = \mathcal{O}(B)$. Assume that C, C' are not hyperelliptic. The curve C' has a $\mathcal{L} \in g_4^1$ only if \mathcal{L} is the pullback of a line bundle of degree 2 on C .*

Proof. The arguments are similar to [Ra, Proposition 2.5, p.234], and we explain them below. Let $\mathcal{L} \in g_4^1(C')$, i.e., \mathcal{L} is a line bundle of degree 4 on C' and $h^0(\mathcal{L}) = 2$. If $\mathcal{L} \simeq i^*L$

then \mathcal{L} descends down to the quotient curve C as a line bundle of degree 2. Suppose \mathcal{L} is not isomorphic to $i^*\mathcal{L}$. Consider the evaluation sequence:

$$0 \rightarrow N \rightarrow H^0(\mathcal{L}) \otimes \mathcal{O}_{C'} \rightarrow \mathcal{L} \rightarrow 0.$$

Since $h^0(\mathcal{L}) = 2$ we see that $N \simeq \mathcal{L}^{-1}$. Tensor the above exact sequence by $i^*\mathcal{L}$ and take its global sections. Since $\mathcal{L} \neq i^*\mathcal{L}$, we observe that $H^0(N \otimes i^*\mathcal{L}) = 0$ and hence $H^0(\mathcal{L}) \otimes H^0(i^*\mathcal{L}) \subset H^0(\mathcal{L} \otimes i^*\mathcal{L})$. In particular, $h^0(\mathcal{L} \otimes i^*\mathcal{L}) \geq 4$. Since C' is non-hyperelliptic, by Clifford's theorem [Hn, IV, 5.4], $h^0(\mathcal{L} \otimes i^*\mathcal{L}) \leq 4$. Hence we obtain the equality $H^0(\mathcal{L}) \otimes H^0(i^*\mathcal{L}) = H^0(\mathcal{L} \otimes i^*\mathcal{L})$.

Now, notice that the line bundle $\mathcal{L} \otimes i^*\mathcal{L}$ has degree 8 on C' and is invariant under i . Hence the product line bundle descends down to C as a line bundle of degree 4. Call this line bundle M . In other words, $\mathcal{L} \otimes i^*\mathcal{L} \simeq f^*M$. Consider the direct image

$$f_*(\mathcal{O}_{C'}) = \mathcal{O}_C \oplus L^{-1}.$$

Hence, by the projection formula, $f_*(\mathcal{L} \otimes i^*\mathcal{L}) = M \oplus (M \otimes L^{-1})$. This gives a decomposition

$$H^0(C', \mathcal{L} \otimes i^*\mathcal{L}) = H^0(C, M) \oplus H^0(C, M \otimes L^{-1}).$$

Moreover, we can identify the eigenspaces for the involution i as follows:

$$(1) \quad H^0(C', \mathcal{L} \otimes i^*\mathcal{L})^+ = H^0(C, M), \quad H^0(C', \mathcal{L} \otimes i^*\mathcal{L})^- = H^0(C, M \otimes L^{-1}).$$

By Riemann-Roch applied to M and $M \otimes L^{-1}$ on C , we get the dimension counts: $h^0(M) = 3$ if $M = \omega_C$, otherwise $h^0(M) = 2$. Furthermore, since C is non-hyperelliptic

$$(2) \quad h^0(M \otimes L^{-1}) = 0.$$

by Clifford's theorem and Riemann-Roch. This implies that

$$(3) \quad H^0(\mathcal{L}) \otimes H^0(i^*\mathcal{L}) = H^0(\mathcal{L} \otimes i^*\mathcal{L}) = H^0(f^*M) = H^0(M).$$

The first equality in (3) implies that the \pm -eigenspaces for the involution i are non-zero. This gives a contradiction to (1) and (2). □

Corollary 2.4. *The generic curve in $\mathcal{R}_{3,2}$ is non-tetragonal.*

Proof. By formula (2) in the proof of Lemma 2.3, the generic line bundle \mathcal{M} of degree 2 on a generic curve of genus 3 has no section. The eigenspace decomposition for the sections of the pullback bundle $\mathcal{L}' := f^*\mathcal{M}$ is given as

$$H^0(C', \mathcal{L}') = H^0(C, \mathcal{M}) \oplus H^0(C, \mathcal{M} \otimes L^{-1}).$$

and which implies that the generic curve (C, B, L) in $\mathcal{R}_{3,2}$ is a non-tetragonal curve. □

2.1. Proof of Theorem 2.1. Consider the inclusion $\mathcal{R}_{3,2} \subset \mathcal{M}_7$ of moduli spaces. Then we recall the classification of the singular loci of the moduli space \mathcal{M}_g done by Cornalba [Co]. In particular, the curves with non-trivial automorphisms lie in the singular locus of \mathcal{M}_g and precisely form the singular locus. The maximal components of the singular locus are also described by him. We recall his result when $g = 7$ and for the embedding $\mathcal{R}_{3,2} \subset \mathcal{M}_7$, since it will be crucial for us. We note that any double cover corresponding to $(C, B, L) \in \mathcal{R}_{3,2}$ corresponds to an involution i on C' with four fixed points, and having the quotient $C = C'/i$.

Proposition 2.5. *The singular locus $\mathcal{S} \subset \mathcal{M}_7$ consists of smooth curves with automorphisms. In particular the moduli space $\mathcal{R}_{3,2}$ lies in the singular locus \mathcal{S} and furthermore it is a maximal component of \mathcal{S} .*

Proof. See [Co, Corollary 1, p.146 and p.150]. □

Now, consider a generic point $(C' \xrightarrow{f} C) = (C, B, \mathcal{L}) \in \mathcal{R}_{3,2}$. Then, by [BCV, §2], we have a decomposition of the canonical space of C' :

$$(4) \quad H^0(C', \omega_{C'}) = H^0(C, \omega_C) \oplus H^0(C, \omega_C \otimes \mathcal{L}).$$

We can also interpret this decomposition for the involution i , which acts on the canonical space nontrivially. Namely, we have a natural identification of the eigenspaces for i :

$$\begin{aligned} H^0(C', \omega_{C'})^+ &= H^0(C, \omega_C) \\ H^0(C', \omega_{C'})^- &= H^0(C, \omega_C \otimes \mathcal{L}). \end{aligned}$$

Note that $\dim H^0(C', \omega_{C'})^+ = 3$ and $\dim H^0(C', \omega_{C'})^- = 4$.

We can now apply Theorem 2.2 to the automorphism i and conclude that i lifts to an automorphism i of \mathbb{P}^{15} and leaves X_{10} invariant. This gives an action of i on the representation space U_{16} . Indeed, since $\text{Pic}(X_{10}) \simeq \mathbb{Z}$, the ample line bundle $\mathcal{O}_{X_{10}}(1)$ is invariant under i . Hence i induces an action on the sections of $\mathcal{O}_{X_{10}}(1)$ which is precisely U_{16} . Let us write the eigenspace decomposition of U_{16} for the i -action:

$$(5) \quad U_{16} = U^+ \oplus U^-.$$

There are various possibilities for the dimensions of U^+ and U^- , which will correspond to

$$(6) \quad (\dim U^+, \dim U^-) := (r, 16 - r), \text{ for } 1 \leq r \leq 15,$$

since i acts nontrivially.

We make the following observation first.

Lemma 2.6. *A point of the product variety $G(3, U^+) \times G(4, U^-) \subset G(7, U_{16})$ corresponds to a linear space $\mathbb{P}^6 \subset \mathbb{P}^{15}$, which is i invariant. Furthermore, if \mathbb{P}^6 intersects X_{10} transversely then the intersection is a non-tetragonal curve with an involution and satisfying the decomposition (4).*

Proof. We first note that a 3-dimensional subspace $V^+ \subset U^+$ and 4-dimensional subspace $V^- \subset U^-$ give a linear subspace $\mathbb{P}^6 \subset \mathbb{P}^{15}$. Clearly $\mathbb{P}(V^+ \oplus V^-) \subset \mathbb{P}(U_{16})$ is a \mathbb{P}^6 and is invariant under the action of i . For the second assertion, note that $C' = \mathbb{P}^6 \cap X_{10}$ also is an i -invariant subset and whenever the intersection is transverse, it corresponds to a genus 7 curve C' (by [Mk]) with an involution, such that \mathbb{P}^6 is the canonical linear system of C' . This means that the \pm -eigenspaces of the canonical space of C' are precisely V^+ and V^- . These data recover the decomposition in (4). \square

Lemma 2.7. *There is a subgroup $H \subset SO(10)$ such that U^+ and U^- are H -representations. This induces an action of H on $G(3, U^+) \times G(4, U^-)$ and which commutes with the action of i such that the group quotient under this action is a birational model of $\mathcal{R}_{3,2}$.*

Proof. We note that by Mukai's classification [Mk, §5], we have a birational isomorphism

$$\mathcal{M}_7 \sim G(7, U_{16})/SO(10).$$

The product subvariety $G(3, U^+) \times G(4, U^-) \subset G(7, U_{16})$ is acted on not by $SO(10)$ but by an algebraic subgroup $H \subseteq SO(10)$. To describe the action of H , we first note that the involution i commutes with the action of H , so that the quotient $(G(3, U^+) \times G(4, U^-))/H$ gives the isomorphism classes of smooth curves with an involution i . Then the matrices in $SO(10)$ which act on the product subvariety are those which commute with the involution i on a linear space $\mathbb{P}(U_{16})$.

As noted in (5), we have an eigenspace decomposition

$$U_{16} = U^+ \oplus U^-$$

for the action of i . Since for any $h \in H$ and $s \in U^+$ (or $s \in U^-$)

$$i.h(s) = h.i(s) = h(s)$$

it follows that U^+ (or U^-) are (projective) H -modules (i.e, are projective representations of H and hence H acts on $\mathbb{P}(U^\pm)$).

By Corollary 2.4, we know that a generic curve $C' \in \mathcal{R}_{3,2}$ is non-tetragonal. Hence, the moduli space $\mathcal{R}_{3,2}$ does not lie in the indeterminacy locus of the birational map

$$\mathcal{M}_7 \dashrightarrow G(7, U_{16})/SO(10).$$

Hence this birational map restricts to a generically injective rational map

$$\psi : \mathcal{R}_{3,2} \dashrightarrow G(7, U_{16})/SO(10).$$

Corresponding to a non-tetragonal curve C' , for $(C, B, L) = (C', i) \in \mathcal{R}_{3,2}$ (which is the generic situation, by Corollary 2.4) we can associate a point in $G(3, U^+) \times G(4, U^-)$ according to the decomposition of the canonical space in (4). Hence the image of ψ maps to the product space

$$\psi' : \mathcal{R}_{3,2} \xrightarrow{\psi'} (G(3, U^+) \times G(4, U^-))/H,$$

and this map is generically injective.

To see that ψ' is birational, given a generic point $\mathbb{P}^6 \in G(3, U^+) \times G(4, U^-)$ we first know by [Mk] that the intersection $C' = \mathbb{P}^6 \cap X_{10}$ lies in \mathcal{M}_7 . Now by Proposition 2.5, C' lies in the singular locus $\mathcal{S} \subset \mathcal{M}_7$, since it has a nontrivial involution. This implies that the inverse image of $(G(3, U^+) \times G(4, U^-))/H$ under ψ in \mathcal{M}_7 is a subset in the singular locus $\mathcal{S} \subset \mathcal{M}_7$ and containing a dense open subset of $\mathcal{R}_{3,2}$. But again by Proposition 2.5 since $\mathcal{R}_{3,2}$ is a maximal component in \mathcal{S} , the inverse image has to be dense in $\mathcal{R}_{3,2}$.

This proves the birational equivalence

$$(7) \quad \mathcal{R}_{3,2} \sim (G(3, U^+) \times G(4, U^-))/H.$$

□

Corollary 2.8. *The moduli space $\mathcal{R}_{3,2}$ (and hence \mathcal{A}_4) is a unirational variety.*

Proof. Since a Grassmannian variety is a rational variety, it follows that the product space $G(3, U^+) \times G(4, U^-)$ is also a rational variety. Using the description in (7), it follows that the moduli space $\mathcal{R}_{3,2}$ is a unirational variety. □

The birational model in (7) should also be compatible with the projection $\mathcal{R}_{3,2} \rightarrow \mathcal{M}_3$. It would be interesting to study H in detail and therefore we pose the following question:

Question 2.9. *Determine the subgroup H and the H -(projective) representations U^+ and U^- explicitly.*

Notice that we have the spinor representation

$$\phi(10) : Spin(10) \rightarrow Aut(U_{16})$$

which gives the $SO(10) = \frac{Spin(10)}{\pm 1}$ -action on $P(U_{16})$, considered in [Mk]. It may be possible to study H further via the spinor representation restricted to the various subgroups of $SO(10)$.

2.2. Towards the motive of $\mathcal{R}_{3,2}$. In this subsection we want to give one application concerning the motive of $\mathcal{R}_{3,2}$.

In [Iy-Ml] we have constructed Chow–Künneth decompositions for open subsets of moduli space of curves of small genus $g \leq 8$. Recall that this was proved in [Iy-Ml], via realizing the open subsets as group quotients of open subsets in homogeneous spaces. The key point used was that the homogeneous spaces have only algebraic cohomology and hence orthogonal projectors equivariant for the group action could be constructed. All those methods can also be applied to the variety $\mathcal{R}_{3,2}$. Using the birational equivalence in (7), we obtain:

Proposition 2.10. *There is an open subset (not necessarily affine) of the moduli space $\mathcal{R}_{3,2}$ which admits a Chow–Künneth decomposition.*

□

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